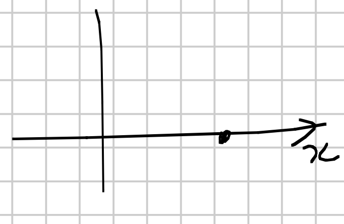
Note Title 19/11/2010

C4 Exercise 2B (solve problems involving parametrics)

1 Find the coordinates of the points where the curve meets the x-axis:

a $x = 5 + t$
$y = 6 - t$



at x-axis $y = 0$ so $0 = 6 - t \Rightarrow t = 6$
hence $x = 5 + 6 = 11$

$\therefore$ coordinates $(11, 0)$

b $x = 2t + 1$

$y = 2t - 6$

at x-axis, $y = 0$ so $0 = 2t - 6 \Rightarrow t = 3$

$x = 2t + 1 \Rightarrow x = 7$

so $(7, 0)$.

1c $x = t^2$, $y = (1-t)(t+3)$

when curve meets x-axis $y = 0 \Rightarrow t = 1$ or $t = -3$

So $x = (1)^2 = 1$ or $x = (-3)^2 = 9$

So coordinates are $(1, 0)$ and $(9, 0)$

1d $x = \frac{1}{t}$ $y = \sqrt{(t-1)(2t-1)}$ $t \neq 0$

$y = 0 \Rightarrow t = 1$ or $t = 1/2$

$\Rightarrow x = 1$ or 2

hence coordinates $(1,0)$ and $(2,0)$

1e $\quad x = \dfrac{2t}{1+t} \qquad y = t - 9 \qquad t \neq -1$

$y = 0 \Rightarrow t = 9$

$\Rightarrow x = \dfrac{2(9)}{1+9} = \dfrac{18}{10} = 1.8$ or $\dfrac{9}{5}$

So coordinates of root are $\left(\dfrac{9}{5}, 0\right)$

2 Find the coordinates of the point(s) where the curve meets the y-axis

a) $x = 2t \qquad y = t^2 - 5$

At intersection with the y-axis, $x=0$

$\Rightarrow \quad 2t = 0 \quad \Rightarrow \quad t = 0 \quad \Rightarrow \quad y = -5$

So coordinates are $(0, -5)$

2b $x = \sqrt{(3t-4)}$ $\quad y = \frac{1}{t^2} \quad t \neq 0$

$x = 0 \Rightarrow 3t - 4 = 0 \Rightarrow t = 4/3$

hence $y = \frac{1}{(4/3)^2} = \frac{1}{(16/9)} = \frac{9}{16}$

So coordinates are $(0, 9/16)$

2c $x = t^2 + 2t - 3$ $y = t(t-1)$

$x = 0 \Rightarrow t^2 + 2t - 3 = 0$

$\Rightarrow (t+3)(t-1) = 0$

$\Rightarrow t = -3, \quad t = 1$

$\Rightarrow y = -3(-3-1) = (-3)\times(-4) = 12$

or $y = 1(1-1) = 1 \times 0 = 0$

So coordinates are $(0, 12)$ or $(0, 0)$

2d $x = 27 - t^3$ $y = \frac{1}{t-1}$ $t \neq 1$

$x = 0 \Rightarrow t = 3$ hence $y = \frac{1}{3-1} = \frac{1}{2}$

So coordinates of y-intercept are $(0, \frac{1}{2})$

2e

$$x = \frac{t-1}{t+1} \qquad y = \frac{2t}{t^2+1} \qquad t \neq -1$$

$$x = 0 \Rightarrow t - 1 = 0 \text{ so } t = 1$$

$$\Rightarrow y = \frac{2(1)}{1^2+1} = 1$$

Hence coordinates $(0, 1)$

3 A curve has parametric equations $x = 4at^2$

$y = a(2t-1) \qquad a \neq 0$

where a is constant. The curve passes through the point $(4,0)$. Find a.

At $(4,0)$ $\quad y=0 \quad$ so $\quad 0 = a(2t-1)$

$\Rightarrow \quad t = 1/2$

Now at $(4,0)$ $\quad x=4 \quad$ so $\quad 4 = 4at^2$

$1 = a\left(\frac{1}{2}\right)^2$

$\Rightarrow \quad a = 4$

4 A curve has parametric equations $x = b(2t-3)$

$$y = b(1-t^2)$$

where b is constant. The curve passes through $(0,-5)$, find b.

At $(0,-5)$ $\quad x=0 \Rightarrow 0 = b(2t-3)$

$b \neq 0$ so $\quad 2t-3=0$

$\Rightarrow t = 3/2$.

At $(0,-5)$ $\quad y=-5 \Rightarrow -5 = b(1-t^2)$

$\Rightarrow -5 = b\left(1-\left(\frac{3}{2}\right)^2\right)$

$\Rightarrow -5 = b\left(\frac{-5}{4}\right)$

$\Rightarrow b = 4$.

5 A curve has parametric equations $x = p(2t-1)$

$y = p(t^3+8)$

where p is a constant. The curve meets the x-axis at $(2,0)$ and the y-axis at A. find p & coords of A.

At $(2,0)$ $y=0$ $\Rightarrow$ $0 = p(t^3+8)$

$\Rightarrow t = -2$

so at $(2,0)$ when $x=2$ $\Rightarrow$ $2 = p(2(-2)-1)$

$\Rightarrow p = -\frac{2}{5}$

now at A $x=0$ $\Rightarrow$ $0 = -\frac{2}{5}(2t-1)$

$\Rightarrow t = \frac{1}{2}$

$\Rightarrow y = -\frac{2}{5}\left(\left(\frac{1}{2}\right)^3+8\right) = \frac{-\cancel{2}^{1}}{1\cancel{5}} \times \frac{\cancel{65}^{13}}{\cancel{8}4} = -\frac{13}{4}$

so A is $(0, -\frac{13}{4})$

6 A curve is given parametrically by the equations $x = 3qt^2$, $y = 4(t^3 + 1)$

where q is a constant. The curve meets